Research-Based Practices for Creating Access to the General Curriculum in Mathematics for Students with Significant Intellectual Disabilities

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# Research-Based Practices for Creating Access to the General Curriculum in Mathematics for Students with Significant Intellectual Disabilities

The objective of this monograph is to increase teacher understanding of creating access to the general curriculum in mathematics for students with significant intellectual disabilities through an exhaustive literature review pertaining to the teaching practices for mathematics instruction for students with significant intellectual disabilities. This monograph will review each of the five components of mathematics recognized by the Council of Teachers of Mathematics (NCTM, 2000). It will discuss how each component of mathematics is addressed and supported by current research for students with significant intellectual disabilities including students who may have additional physical and sensory impairments. A description of the findings consisting of the mathematical practices and the supporting evidence will provide specific instructional strategies for teaching mathematics for students with significant intellectual disabilities for educators.

## Background on Access to the General Curriculum

Access to the general education curriculum for all students is not a school or district choice; it is mandated by law. According to The Individual with Disabilities Education Act (IDEA, 1997, 2004), all students with disabilities must access the general education curriculum to the maximum extent possible. Accountability for the education of children with disabilities is mandated by the No Child Left Behind Act (NCLB, 2002). NCLB requires the participation of all students with disabilities in both state and district assessments in the areas of English/Language Arts (ELA), math, and science.

Both IDEA and NCLB make is it necessary for educators to find the most effective strategies and practices to teach students with significant intellectual disabilities and assess learning outcomes for academics. Practitioners need a model for prioritizing standards and instructing students with significant intellectual disabilities to access grade-appropriate content in the general education curriculum areas of ELA, mathematics, and science. However, one of the challenges facing educators in regards to providing effective strategies in reading, math, and science to students with significant intellectual disabilities is the absence of a clear conceptual framework.

## National Council of Teachers of Mathematics and National Mathematical Components

The National Council of Teachers of Mathematics (NCTM, 2000) recognizes five national mathematical components and provides a set of standards to address the achievement of all students.  The five national components of mathematics are number/operations, algebra, data analysis, geometry, and measurement. However, Browder, Spooner, Ahlgrim-Delzell, and Wakeman (2008) found that there is limited research about teaching mathematics to the standards of algebra, geometry and data analysis to students with significant intellectual disabilities (i.e., students with moderate or severe disabilities that may have other disabilities such as autism or physical and sensory impairments). The results of a meta-analysis teaching math to students with significant intellectual disabilities conducted by Browder, et al found that numbers/operations and measurement were the primary national mathematical components of instructional focus for this population. The limited information available to educators about skill priorities, effective instructional strategies, and measuring outcomes for mathematical instruction for students with significant intellectual disabilities across the five mathematical components creates an urgent need for research to address these issues.

The NCTM (2000) recognizes the uniqueness of all students and advocates for the instruction of all five components of mathematics as a life-long approach to for the on-going development of reasoning skills and effective problem-solving strategies to be utilized in a personally relevant manner on a daily basis. In addition to compliance with the federal mandates for educated students with disabilities, mathematical instruction across the five national components creates opportunities to develop essential skills for students with significant intellectual disabilities in the areas of independent functioning, quality of life, and self-determination. Practical application for all students can be found in each of the five national mathematical components discretely or in overlap; therefore, the omission of any national mathematic components from instruction will result in fragmented knowledge and limitations in application (National Mathematics Advisory Panel, 2008).

Individuals will find occasion to encounter all five components of the national mathematical standards throughout their lifetime (National Mathematics Advisory Panel, 2008). Numbers and Operations provide a foundation for other four national mathematical components. The development of number sense is essential because numbers are encountered in everyday life. Daily application includes examples such as street address, clothing size, sports scores, and cooking. The manipulation of numbers through operations provide a means for developing a budget, balancing a checkbook, or estimation. Algebra knowledge provides the opportunity to analyze the relationships between variables. This information is helpful when shopping for clothes, as there is a relationship between clothing size and the height or weight of an individual. The concepts learned in geometry give the learner an opportunity to practice skills essential to navigate their surroundings. Data Analysis assists individuals to developing self-determination such as creating goals and graphing progress.  Everyday uses in measurement include chores and interests such as cooking, decorating, and gardening.

## Teaching Mathematics to Students with Significant Intellectual Disabilities

Various studies have synthesized the instructional content for students with cognitive disabilities in the areas of mathematics (e.g., Mastropieri, Bakken, & Scruggs, 1991; Xin, Grasso, Dipipi-Hoy, & Jitendra, 2005). The primary focus of instructional content for the studies includes basic skills, rule learning and problem-solving, and applications, such as skills associated with the time, money and measurement. The reviews on the other national components of mathematics, such as algebra, usually included student participants with learning disabilities or other high incidence disabilities (Jitendra & Xin, 1997; Maccini & Gagnon, 2000; Maccini, McNaughton, & Ruhl, 1999; Miller, Butler, & Lee, 1998; Xin & Jitendra, 1999).

The meta-analysis of studies teaching math to students with significant intellectual disabilities conducted by Browder, Spooner, Ahlgrim-Delzell, Harris, and Wakeman (2008) sought to identify evidence-based practices for teaching mathematics specifically for this population of students. It organized the literature based on the five national components of mathematics. The analysis identified 54 single-subject studies and 14 group studies for a total of 68 studies from 1975-2005.

A comprehensive review of the literature from this meta-analysis found that mathematics instruction occurred most often in the areas of numbers/operations and measurement but found 6 studies in algebra, 3 studies in geometry, and 2 studies in data analysis for this student population (Browder, et al., 2008). This synthesis of the literature indicates the need for more research about teaching mathematics inclusive of algebra, geometry, and data analysis in ways that are effective and personally relevant for students with significant intellectual disabilities. Although the number of studies is limited in these three components of mathematics, the results of these studies illustrate that they can learn skills across the five national components of mathematics, not just the numbers/operations and measurement components.

This monograph will provide professionals in the field of special education an exhaustive resource on research in mathematics for this population as well as research-based practices needed to develop the mathematical skills for students with significant intellectual disabilities. A thorough discussion of teaching practices will address instructional methodology, the use of assistive technology, and instructional planning for students with physical or sensory impairments when teaching mathematics across the five national components.

## Method

### *Literature Search Procedures*

This literature review attempted to locate the most current research (i.e., quantitative, qualitative, correlational) from peer-reviewed journals in special education, psychology, and research in which a mathematics skill was taught to at least one individual with significant cognitive disabilities. A total list of 33 search terms describing both the population (e.g., moderate mental retardation, severe disabilities, autism), was used in combination with terms related to instruction in mathematics (e.g., money, counting, graphing, geometry, measurement). For example, the term severe disability was used in combination with each of the mathematical instruction terms. Both an electronic and a hand search were conducted to determine articles for review. Electronic databases searched included InfoTrac, Masterfile Premier, ERIC, PsychInfo, and Academic Search Elite. In addition, authors hand searched articles pertaining to the field of special education from 2003 until April 2009 in the following peer reviewed journals: *Education and Training in Mental Retardation and Developmental Disabilities, The Journal of Special*

*Education, Focus on Autism and Other Developmental Disabilities, Research and Practice for Persons with Severe Disabilities,* and *Journal of Applied Behavior Analysis.*

Inclusion and Exclusion Criteria

In order for a study to be included in this literature review it needed to meet the following criteria: (a) original research that included an intervention which focused on teaching an academic mathematics skill even if not the focus of the study; (b) inclusion of at least one individual who could be classified as having a significant intellectual disability (defined as having an IQ of 55 or less and/or a description of the student as having moderate, severe or profound disability); (c) inclusion in a peer reviewed journal published between 2003 and May 2009; and (d) the intervention produced measureable results describing student outcomes. Since the authors wanted an exhaustive review of all recent research literature pertaining to mathematics instruction for students with significant cognitive disabilities, we used the following criteria to determine inclusion based on research type: (a) for both single subject and group studies, the design needed to be either experimental or quasi experimental, (b) for correlational studies, the relationship between a mathematical skill/ability and other variables needed to be examined, and (c) for qualitative studies, academic outcomes for students in the area of math needed to be addressed within the research question, or an emergent theme derived from data analysis. Reviews of the literature that synthesized previously conducted research and dissertations were not included in the review. Following these rules, some studies of mathematics interventions were not included in this review. For example, a qualitative study by Hunt, Soto, Maier, and Doering (2003) was excluded because the results focused on social outcomes for students in relation to an intervention focused on accessing general education curriculum, but the academic outcomes for the students were not presented. Another single-subject study conducted by Mechling, Gast, and Cronin (2006) was excluded because it studied the fluency of task completion for three chained tasks. Some of the tasks in the chain were not math-related. The results for the math-related tasks could not be separated from the non-math-related tasks. Finally, a randomized control group study by Shen (2006) was excluded because of inappropriate analysis of data, it reported multiple t-tests only for post-test data of the experimental and control groups. An ANOVA that incorporates pre- and post-test for both groups is the appropriate analysis for this type of randomized control design.

## Coding

Once an individual study met the inclusion criteria, a literature review coding form was developed to record the characteristics of the study for further analysis. These characteristics included (a) mathematics academic component, (b) targeted mathematical skill, (c) level of alignment with general curriculum, (d) research study design type, (e) participants, (f) level of cognitive engagement, (g) assistive technology, (h) format of instruction (e.g., type of prompting and prompt fading), (i) other outcome measures (i.e., generalization, maintenance, and social validity), and (j) results. Inter-rater reliability was used to check for coding agreement for one-third of the review forms using a point-by-point method. Number of agreements were divided by total number (agreements plus disagreements) of coded items, then multiplied by 100 to convert it to a percent.

Following coding, individual studies were entered into a statistical database program (SPSS, 2004). For descriptive analysis of the studies, frequencies and types of each of the following were calculated (a) NCTM component of mathematics, (b) targeted math skill, (c) instructional format (i.e., one to one), (d) instructional procedure, (e) prompting strategy, (f) error correction, (g) reinforcement, (h) setting, and (h) individual teaching. Each dependent and independent variable was entered separately when studies used either multiple dependent or independent variables. For example, McDonnell, et al. (2006) used an alternating treatment design to compare one-to-one embedded instruction in a general education setting to a small group format in special education setting. The embedded instruction used constant time delay, differential reinforcement, and error correction while the small group instruction employed an intrasequential format with spaced trails (e.g., using a round robin style). In this example, each independent variable would be coded. Further, for studies in which the dependent and/or independent variable was related to training of peers, parents, or teachers, only the variables directly affecting student outcomes in mathematics were considered. For example, in a study conducted by DiPipi-Hoy and Jitendra (2004), independent variables included parent training using targeted skill selection and instructional planning as well as constant time delay to teach their children steps of a purchase skill. In this case, only the independent variable for constant time delay was coded.

# Results

## Inter-rater Reliability

Three researchers came to consensus for the inclusion of the articles, no disagreements occurred. Inter-rater reliability was conducted on 28% of the articles. Overall inter-rater reliability for coding the article components was 96.8% with a range of 95-100%.

## Description of the Included Studies

Overall, 17 articles were located and reviewed to decide the extent in which each met the mathematics inclusion criteria. A total of three articles were excluded from the research literature review. Two articles were excluded because the targeted learning objective could not be classified as relating to mathematics. One article was excluded because none of the participants were classified as having a moderate or severe disability. The remaining fourteen articles summarized in Table 1 were located in five different publications met the inclusion criteria and were included in the analysis. All of the identified 14 studies were conducted using single subject design. No studies were found using group, nonexperimental/correlational, or qualitative research designs. A multiple probe research design was implemented for eight of the studies, a multiple baseline design was used in two studies, and an alternating treatment design was used in four studies.

All of the 14 studies provided evidence of increased student outcomes. Although analyzing single-subject data is controversial, in order to describe the pattern of student growth for the studies in this review the Percent of Non-overlapping Data (PND) was conducted. In this procedure data from the treatment phase is compared to the baseline phase and all data in the treatment phase that overlaps (is lower) than the highest baseline data point is eliminated. The remaining number of data points above the highest baseline data point is divided by the total number of data points in the treatment phase. PND values can give a rudimentary description of student growth resulting from the instruction. The PND values of the studies included in this review ranged from 46% to 100% with a mean of 87.52% and a median of 97.92%. This means that the students increased their rate of correct responses from 46% to 100%. A 100% PND means that all of the student responses during the treatment exceed their highest response rate in the baseline phase before the instructional intervention began. A PND of 46% means that more than half of the responses during the intervention were below baseline performance. Even though this is an increase in correct responding it is considered very small.

A total of 47 individuals with disabilities participated in these studies; however only 37 of the participants were identified as students with significant intellectual disabilities. Only the information pertaining to the students with significant cognitive disabilities was coded and analyzed. Of those 37 students, 24 were male and 13 were female. Participant ages ranged from 7 – 22 years old, with two studies conducted in elementary, five studies in middle grades, and eight studies in high school. One study was conducted in both middle and high school grades. Twelve students with moderate disabilities, three with severe/profound, two with autism, two with developmental disabilities, and two with multiple disabilities participated in included studies.  Four of the studies included students with physical disabilities and only three studies were found that included a student with sensory impairment (Ayres et al., 2006; Collins, 2007; Polychronis et al, 2004). Ethnicity of participants was not noted in any of the 14 articles. The majority of studies were conducted with students on a symbolic level of communication; however, four studies were found with students at an early symbolic level, but only two studies were found with students functioning at a presymbolic level. Three studies were conducted with students at multiple levels (i.e., two or more) of communication.

Research was found in four of the five components of mathematics outlined by NCTM; no studies were found demonstrating target skills in data and probability. Nine studies were conducted on skills that fall in the strand of Measurement, one in Geometry, four in Algebra, and 11 in Numbers and Operations. Nine of the 14 studies included target math skills that fell within more than one strand of math. The level of alignment with the general curriculum found all skills to be academic using age appropriate materials with scoring rubrics based on independence with minimal assistance and potential barriers were minimized when appropriate; however, only six of the 14 studies focused on target skills that were referenced to the student's assigned grade level and focused on achievement that maintained fidelity with the content of the original grade level standards. Levels of cognitive engagement (Bloom, 1956) were examined finding all 14 articles targeting skills that fell within the *remembering* level, only 4 within *understanding*, and 4 within *applying*. No studies were located that included target skills in the levels of *analyzing*, *evaluating*, or *creating*.

Some form of assistive technology was used in eight of the studies. Four studies incorporated low tech devices that required no plug or batteries such as a number line or touch points. Five studies incorporated low high tech devices including computer assisted instruction and an adapted mouse. One study used both a low and high tech assistive device. No studies were found using assistive technology for communication purposes.

Nine of the fourteen studies were conducted using one-to-one instruction, four with small group instruction, and two with whole group. One study was conducted using both small and whole group instruction. All fourteen studies were conducted using systematic instruction, including two that used milieu teaching, 11 that used with practice with feedback, seven that used total task chaining, eight that used massed trials, one that used distributed trials, and two that used embedded trials. No studies were found that used stimulus prompting (e.g., stimulus shaping, stimulus fading). Response prompting was used for all studies. Specifically, 12 used verbal directions, seven used a verbal model, four used physical models, and one used physical guidance. There were no studies that used a picture cue. Thirteen studies used response fading prompts. Specifically, five used simultaneous prompting, three used a most-to-least prompt hierarchy, two used a least-to-most prompt hierarchy, and four used a constant time delay procedure. Praise was used in all 14 studies as the primary reinforcement for correct answers, two studies also provided access to tangible materials as reinforcement as well.

Ten of the 14 studies were conducted in the special education classroom. Two of the fourteen studies were conducted in both the general education and special education classroom

(Collins et al., 2007; Jimenez et al., 2008) and one study was conducted in both the special education classroom and community (Cihak, et al., 2006). Only one study was found that was conducted solely in the general education classroom (Polychronis et al, 2004). In seven of the studies the special education teacher taught the target skill. A researcher taught the target skill in three studies and a general education teacher taught the skill in one study (Collins et al., 2007). Peers taught the target math skill in one study, the parent taught in one study, and the para-educator taught in one study. No studies were found in which the student self-taught or directed their own learning. Two studies used computer assisted learning to teach all targeted skills (Ayres et al., 2006; Hansen & Morgan, 2008).

Finally, 13 of the 14 studies conducted generalization measures on targeted skills, specifically nine across settings, two across people, and ten across materials. Maintenance data was collected for ten of the studies, spanning from one week to one month after mastery. Social validity was collected for only six of the 14 studies.

## Description of Teaching Practices

From this review eight important teaching practices were identified. For each of the eight identified teaching practices a description of the practice and evidence from the literature supporting the practice is provided.

### Mathematics Practice 1: Use of Systematic Instruction across Strands in Math

Teachers can consider the use of systematic instructional procedures to teach math skills to students with significant cognitive disabilities across the strands of numbers and operations, measurement, geometry, data analysis and probability, and algebra to be an evidence-based practice (Browder, Spooner, Ahlgrim-Delzell, Harris, & Wakeman, 2008).

*Description of the Practice*

One of the strongest practices for teaching math across the strands to students with significant cognitive disabilities is the use of systematic instructional procedures (Browder, Spooner, Ahlgrim-Delzell, Harris, & Wakeman, 2008). Systematic Instruction has been described by Snell (1983) as an instructional method following an organized and predicable format that demonstrates currently acknowledged best practices and uses performance data to drive instructional changes. Depending on the targeted skill selected for instruction, practices that fall under the umbrella term systematic instruction may include using a response chaining procedure to teach chained tasks or a time delay strategy to teach discrete tasks. Discrete skills are those that consist of a single step, have a definite beginning and ending, and can be counted as correct or incorrect. In contrast, chained skills are a series of discrete steps that are linked together sequentially to form a targeted skill (Snell & Brown, 2006). Teachers using a flashcard format to teach numerals or fractions would be targeting discrete skills, while students learning the procedure for a two digit addition problem would be learning a chained task. Discrete skills can be taught in the following formats: (a) massed trials (e.g., one flashcard after another); (b) distributed trials (e.g., asking the student to tell time on the clock throughout the day); or (c) embedded trials (e.g., in a general education setting, a teacher or peer asks the student to respond to “Show me point A on the coordinate plane” throughout the geometry lesson). Chained tasks may be taught using the following presentations: (a) total task (e.g., the student practices the all of steps in a long division problem every time); (b) forward chaining (e.g., the student learns the steps of a task analysis for data analysis one at a time; first he/she learns to read the problem, then he/she learns to find the math facts, and so on); or (c) backward chaining (e.g., the student completes the steps of a purchasing sequence, mastering paying the cashier, which is the last step, before learning the step before paying the cashier).

*Evidence*

Evidence was found in all 14 research studies in support of using systematic instruction to teach targeted skills across all math strands found. Seven studies used total task chaining to teach math skills. In one example Jimenez, Browder, and Courtade (2008) taught students to complete a nine-step task-analysis to solve an algebraic equation. Another method of instruction that was widely used among the studies was the use of massed trials to teach a variety of skills, for example time telling (Falkenstine, Schuster, Kleinert, 2009); addition (Neef, Nelles, Iwata, & Page, 2003); and multiplication facts (Rao & Mallow, 2009).

Findings from this review are consistent with prior math reviews (Browder et al., 2009) in regards to the use of systematic instruction to teach math skills to students with significant disabilities. All 14 studies used systematic instruction to teach math skills to mastery, due to overwhelming support of this practice, it can be recommended as an instructional strategy to use in teaching mathematics.

**Mathematics Practice 2: Functional Applications and Teaching in the Context**

Functional math skills are enhanced when teachers encourage students to apply them in real world contexts. “Teaching opportunities that are embedded in the context of meaningful social interaction between partners with shared interests are most likely to result in learning new, functional communication forms” (Snell & Brown, 2006, p. 455).

*Description of the Practice*

Teaching and practicing math skills in a community setting, in the school cafeteria, or in a vocational setting can create functional, real world applications of acquired content for students with significant disabilities. For example, in a grocery store setting, students may learn the following mathematical skills: (a) how to "map out" their route based on their grocery list (a skill linked to geometry); (b) how to make purchases based on the steps in a task analysis for purchasing skills, or the next dollar up strategy to count out money (measurement skills); and/or (c) how to tell how much time they have to shop (another measurement skill). In this way, the grade appropriate mathematical skills the student has learned in the classroom directly correspond to skills they will need throughout their lives. Further, if teachers use opportunities throughout the day to teach mathematical skills, the chances that the student can learn and apply the information are greater. For example, if a class is learning math within the strand of algebra, their knowledge of how to solve an algebraic equation may be demonstrated by using a linear equation to answer a question that arises during a literacy lesson, to figure out the age of a person in a biography when they died (Year Born + X = Year Died).

*Evidence*

Only two studies were found in which the targeted skill was taught in the context of the actual activity (milieu). Polychronis et al (2004) taught students to identify numbers and tell time within daily activities during the classroom routine, and Collins et al. (2007) taught students to identify sight words within the daily routine. Only two studies, both targeting purchasing skills, were taught within the community (Cihak et al., 2006; DiPipi-Hoy et al., 2004). While evidence from this literature search does not exemplify strong support of using milieu teaching and the community as a setting for practice, this does not mean that educators should not continue to use these practices. More evidence is needed in which students are taught math skills within the natural context it is to be used.

Evidence from this review did show some support in using personally relevant tasks to target grade-level appropriate math learning objectives. Jimenez et al. (2008) taught high school students with moderate disabilities to solve an algebraic equation to complete work related tasks. Students were asked to find "how many more" they needed to complete the work task independently. Another example of using mathematics to teach functional skills was demonstrated by Mechling et al., (2008) in which participants were taught to follow a task analysis to complete cooking tasks. Specific steps of the task analysis involved mathematics skills such as identifying numbers on a microwave, stirring 6 times, gathering a specific number of food materials, and measuring food items (e.g., ¾ cup). In addition, studies were found that taught personally relevant skills such as purchasing and time telling (Falkenstine et al., 2009; Hansen & Morgan, 2008; Polychronis, et al., 2004).

One strategy for personal relevance within all mathematics strands across the grade levels is to continue to use in vivo teaching and determine the essence of a mathematics strand before teaching it (e.g., why do we learn about coordinate planes in geometry?). Once the reason behind the standard is determined (e.g., develop of path to be followed, applied with a map), naturalistic teaching with personal relevance can be determined.

Previous literature has demonstrated a strong effect for in vivo teaching, including milieu teaching (Snell & Brown, 2006, p. 469), community based instruction (Browder & Spooner, p.251), and incidental teaching (Snell & Brown, p.463). Further, a meta-analysis conducted on teaching mathematics to students with significant cognitive disabilities by Browder, Spooner, Ahlgrim-Delzell, Harris, & Wakeman (2008), found strong support for in vivo instruction, as all of the studies they reviewed had some applications of the mathematical skill to a real world context (e.g., store, restaurant). Considerations for functional applications of core academic content (e.g., math) should include a “functional task” meaning that it is useful, but should also be applied within the natural context that the objective would be used. Functional applications for math may include examples such as using a keypad on a microwave, or locating a house by its house number (Snell & Brown, p. 499) or following a daily classroom schedule using time (Browder & Spooner, p. 232), but should not be limited to skills typically considered “functional.” Solving a prediction problem to indicate how likely it is to rain tomorrow, using a linear equation to indicate how much money is left to spend at the mall, developing a coordinate plane to find the shortest path in grocery shopping, or graphing movie preferences to decide which movie to rent are all examples of math skills that can be taught within the natural context of the event and demonstrate a strong link to national and state standards. Caution should be made for educators to not limit students learning to skills that have typically been taught in math for this population, but to rather expand student’s mathematical reasoning to include deeper applications of problem solving across natural contexts.

**Mathematics Practice 3: Use of Appropriate Prompting and Fading**

Teachers need to consider the mathematical skill being taught when choosing the prompting strategy. In addition, fading of any prompting strategy is imperative for the student to reach mastery independently.

*Description of the Practice*

Stimulus prompts (also called antecedent prompts) typically involve the modification of the materials whereas response prompts are actions carried out by the instructor (Collins, 2007). Stimulus prompting strategies involve stimulus shaping and stimulus fading. For example, when teaching the concept of a circle, a teacher might use stimulus shaping by first having the student discriminate between a circle and a rectangle. Over a series of trials, the rectangle would be rounded out to create an oval. By the end of the instructional trials, the student is able to discriminate between a circle and an oval by recognizing the relevant difference between the oval and circle is the length, versus thinking they are both circles just because they both are round. Stimulus fading is when “the instructional materials highlight an irrelevant feature to facilitate the success of the student” (Collins, 2007, p. 126). For example, a student might know how to count objects, but not have numeral recognition. At first, the teacher presents the numeral three with three objects embedding in the numeral itself (e.g., three apples on each of the points of the numeral three). Then, the apples are faded into points. Over successive trials, the irrelevant information (i.e., the points) is faded and only the salient information (i.e., the numeral) remains.

Response prompting strategies include graduated guidance, most to least prompting, system of least prompts, time delay and simultaneous prompting. All of these strategies begin with securing attention and delivering the task direction. In graduated guidance, the student is provided with physical assistance as needed while the teacher shadows them. Most to least prompting starts with the most intrusive prompt (e.g., full physical prompt), and then over time is systematically faded to less intrusive prompts. The system of least prompts is the opposite; the least intrusive prompt is delivered first and if the student does not independently respond, the next prompt from the prompt hierarchy is given. Time delay is considered an errorless learning procedure in that the controlling prompt and the task direction are delivered simultaneously for the first trials. For future trials, the controlling prompt is delayed after a specific time interval. Finally, simultaneous prompting occurs when the controlling prompt and task direction are delivered together and fading of the prompt does not occur (Collins, 2007).

*Evidence*

All fourteen research studies located in this review used response prompting to teach targeted math skills. Strong evidence for using response prompting (graduated guidance, most to least prompting, system of least prompts, and constant time delay) was demonstrated in this review. Specifically, three studies were found that used constant time delay, three additional studies used most to least prompts, and two additional studies used the least to most prompting system. Jimenez et al. (2008) used both a least to most prompt system and constant time delay to teach algebra. Five studies did not fade the prompts, demonstrating simultaneous prompting procedures, the issue using this procedure is the limited ability for students to master skills independently.

With independent responding to be the primary objective for student's mastery of target math skills, constant time delay and prompting hierarchies demonstrate strong evidence in the literature for teaching math skills. Both prompt systems are widely supported in the literature of teaching students with significant disabilities (Collins, 2007; Snell & Brown, 2006). Specifically, a literature review on the use of time delay to teach functional and academic skills was recently conducted by Browder, Ahlgrim-Delzell, Spooner, Mims, and Baker (2009) demonstrating time delay to be an evidence-based instructional procedure. While stimulus prompting (fading and shaping) was not used to teach math skills in this review it should also be considered.

**Mathematics Practice 4: General Educator as Content Specialist**

The general education teacher can serve as a resource for ensuring that the mathematical content for students with significant disabilities aligns with the state standards (Snell & Brown, 2006). Special and general education teachers can collaborate during instructional planning so that the mathematical concepts are both critical and personally relevant for all students.

*Description of the Practice*

According to the “highly qualified” requirement from the No Child Left Behind Act (NCLB, 2002) teachers should be skilled and knowledgeable in the content area being taught. This may require having a deeper understanding of mathematics than special education teachers have received in their teacher preparation programs. When special education teachers start planning lessons for their students in the areas of geometry, algebra, or data analysis, they may ask themselves “Is this really algebra? Or have I veered too far off of the grade level standard?” They may also be faced with the question “Why do students need to know these mathematical concepts?” While the special educator is skilled at adapting the general education lesson for his/her students to ensure participation in a meaningful activity, a general education teacher can help the special education teacher with concerns of alignment and the big ideas of the lesson. Together, the teachers can review lessons to address the following criteria for alignment to the general education curriculum as suggested by Flowers, Wakeman, Browder, and Karvonen (2009): (a) the content is academic and includes the major domains/strands of the content area, (b) the content is referenced to the students’ assigned grade level, (c) the focus of achievement maintains fidelity with content of the original grade level standards, (d) most of the materials are age appropriate, (e) scoring rubrics focus on student performance with minimal assistance, and (f) potential barriers to demonstrating what students know and can do are minimized.

*Evidence*

In this review all fourteen studies taught academic content aligning to the major strands of math, used age appropriate materials, used scoring rubrics based on student performance with minimal assistance, and controlled for potential barriers. While all target skills found were aligned to mathematical strands defined by NCTM, only six were aligned to the participants grade-level and maintained fidelity to the intended standard (e.g., was the skill “really” aligned to the geometry skill in middle school?). For example, Cihak and Grim (2008) compared the use of number lines and touch points to teach elementary students to solve simple addition problems. The targeted skill of addition was aligned to the math strand of Numbers and Operations and maintained fidelity to the grade-level standards. The fact that six of the studies found exhibited this high level of alignment to the general curriculum shows great promise of content alignment due to recent legislative mandates.

According to Browder et al., (2008), there has been limited evidence demonstrating math instruction for this population in other content areas than Numbers and Operations, and Measurement. Even within those strands of mathematics, targeted skills have generally not linked to grade-level expectations and rarely aligned to the fidelity of the standards (e.g., number identification in high school, over representation of money and time skills in measurement across all grade-bands.) The evidence found from this literature study was consistent with such finding, but did show development in the literature surrounding teaching math skills to this population. This review found 4 additional studies teaching algebra, and 1 in geometry. Using the alignment criteria set by Flowers et al., (2009), more research is needed aligning NCTM math strands using best practices.

**Mathematics Practice 5: Teaching Math Skills in the General Education Setting**

Some authors would suggest that in order for a student to have access to the general curriculum, they must first be present in a general education classroom (Wehmeyer & Agran, 2006). According to Wehmeyer and Agran, "if students with significant cognitive disabilities are to achieve access to the general curriculum, it will be in the context of the general education classroom with students receiving the supplementary aids and supports, high quality instruction, and curriculum modifications they need to succeed (p. 20)."

*Description of the Practice*

Evidence suggests that students can learn academic tasks in general education settings (math vocabulary, Collins et al., 2007; algebra equation, Jimenez et al., 2008; number identification and time telling, Polychronis et al., 2004). In addition, research shows that observational learning can occur as a result of students with disabilities inclusion in heterogeneous groups. Further, the skills learned in a general education math class may not be limited to math facts alone, but may include other skills addressed on the students' IEPs, such as reading, writing, communication, social, and motor skills (Collins, Kleinert, & Land, 2006).

The question for many teachers is how to address functional mathematic skills, align these to state standards, and to accomplish this is in a general education setting, especially when the general education curriculum is a barrier for many students. To address the barriers from the curriculum that many students (not only students with significant cognitive disabilities) encounter, a framework has been proposed called "Universal Design for Learning" (CAST, 2007). UDL abandons the historical notion in our schools that the curriculum is unattainable by some students due to *student* deficits, and shifts the focus of deficits to the *curriculum*. In the past, special education teachers have made accommodations and modifications to existing curricula. What if the curriculum were designed to meet the needs of diverse learners in the beginning? When the deficits or barriers of the curriculum are addressed from the planning stages to meet the needs of all learners in the classroom, there is a reduced need to retrofit (e.g., accommodate/modify) the curriculum for certain students. According to the UDL framework, curriculum goals, materials, and assessments can be presented addressing multiple means of representation, expression, and engagement, thereby reducing the need to individualize for each student in the classroom.

*Evidence*

Despite the recommendations from the literature on collaboration for inclusion in general education settings, including the framework of UDL, few models currently exist for how to teach mathematics to students with significant cognitive disabilities with their typical peers. This review only found three examples of math instruction that occurred in the general education setting, and only two of the three demonstrated instructional models in which the general education teacher taught targeted math skills (Collins et al., 2007; Polychronis et al., 2004). Specifically, both Collins et al., (2007), and Polychronis et al., (2004) used embedded instruction to distribute math trials within the natural context of the lesson. For example, in Collins et al. (2007) functional and core content sight words (e.g., angle) were embedded throughout the math lesson. The general education teacher was able to embed trials for the students to identify sight words while the math lesson was occurring, rather than implementing massed trials at a less appropriate time.

**Mathematics Practice 6: Use of Assistive and Other Technologies**

Planning for mathematical instruction needs to include the use of high and low technologies and communication devices to modify to content, materials, activities, or mode of student response to ensure accessibility for all students.

*Description of the Practice*

The impetus for the development of assistive technology was for individuals with cognitive, physical, and sensory impairments that required assistance to perform both academic tasks and nonacademic tasks (Silver-Pacuilla & Fleischman, 2006). Students with significant intellectual disabilities may require the use of assistive technology as the ‘floor of opportunity’ to access and achievement (Parette & Peterson-Karlan, 2007). Teachers must possess knowledge about the availability and suitability of assistive technology options for mathematical instruction. Items such as number lines, graphic organizers, jigs, and speech recognition software are examples of assistive technology to use during mathematical instruction. For example, the use of speech recognition software to express answers may be an appropriate option for the student that has limited use of arms and hands.

Teachers must consider the role of communication systems for their students during mathematical instruction, so they will need to think about how students will express information such as definitions of key concepts and solution to problems. Students with significant intellectual disabilities often require the use of augmentative or alternative communication to meet their communication needs in during social interaction and academic instruction, so different communication systems may be utilized in different context (Downing, 2001). Teachers will need to ensure that communication systems are an appropriate match for target responses during mathematical instruction to increase opportunities for students to participate and express what they have learned (Hourcade, Pilotte, West, & Parett, 2004; Sigafoos & Iacono, 1993).

*Evidence*

Consistent with the limited literature to support assistive technology with core academic skills, this review found examples of high and low tech technology being used with students. However, the instructional reasoning to support the use of a number line (low tech) or keyboard for number identification (high tech) was sparse. Additionally the uses of assistive technology generally responded to the needs of the skill being taught rather than the individual needs of the student that was being taught. While consideration for appropriate materials to be used when teaching math skills should be made, additionally considerations need to be made for the needs of the individual student regarding assistive technology. Using the alignment criteria set by

Flowers et al., (2009), there was no evidence from this review that barriers were not met; however, very limited student information was given for most studies. More research is needed in the area of using assistive technology to teach mathematics, with special attention to meeting the diverse communication needs of students with significant disabilities.

**Mathematics Practice 7: Training for Fluency, Maintenance, and Generalization of Skills**

In teaching mathematical skills, teachers should focus on the four phases of learning for students in all grade levels: (a) acquisition, (b) fluency, (c) maintenance, and (d) generalization (Wolery, Ault, & Doyle, 1992).

*Description of the Practice*

Most teachers address acquisition which is the initial learning of a new skill. Fluency is how quickly and easily the student performs the acquired skill. Maintenance is how the skill is sustained over time, while generalization is how the student can use the skill across settings, with different people, and with a variety of materials. For math, a student may acquire multiplication facts initially through systematic instruction and repetition of the skill. The teacher can measure fluency by quizzing the student on how many facts he/she can do in three minutes. As the student acquires new skills over the next few weeks or months, the teacher measures maintenance of the multiplication facts periodically and re-teaches if the student has not retained the information. Finally, the teacher ensures generalization by having the student use the multiplication facts (a) in the classroom, at home, in the community, (b) with teachers, peers, and parents, and (c) with worksheets, flashcards, and real world applications. When teachers address all phases of learning, the likelihood that the skill will be used across the student's life time is increased.

*Evidence*

All fourteen studies found acquisition of target skills, in addition strong evidence of generalization was shown by thirteen of the fourteen studies. Generalization across materials was the most common measure found, with nine studies showing generalization across settings (special education classroom, general education classroom, community settings, and other school settings). For example, Jimenez, Browder, & Courtade (2008) generalization data was collected across settings, materials, and people. To assess generalization across settings, data were collected in both a special education and a general education algebra classroom; to assess generalization across materials, students were able to generalize the materials used for the algebraic equation to other materials, such as spoons, pencils, pens, and candy. Finally, to determine generalization across people, the special education teacher as well as a typical peer conducted the algebra intervention with the students.

**Mathematics Practice 8: Considerations for Students with Sensory and Physical Impairments**

Teacher should understand the potential barriers to access for students that have sensory and physical impairments by focusing on various ways to present instructional content and represent materials. Teachers also need to identify modes of response and learning activities for students that are appropriate matches to ability and interest.

*Description of the Practice*

The Center for Applied Special Technology (CAST, 2008) conceptualized Universal Design for Learning Center (UDL) as a model for curricular and instructional planning to meet the unique needs of all students including those with sensory and physical impairments. The educational approach of UDL encompasses three principles of learning that include representation, action, and engagement (CAST, 2008). UDL provides an effective guideline for teachers to reference for the daunting challenge of preparing to meet the diverse needs of all students that will expand opportunities for learning and access to the general education curriculum.

UDL makes the seemingly impossible task of meeting the needs of all students in the classroom a possibility (Howard, 2004). This model embraces and curricular flexibility and student differences. UDL provides educators with the ability to plan numerous ways for students to access academic content, show what they know, and become actively engaged in the learning process. When planning mathematic instruction for students that has sensory and physical impairments, the teacher can modify materials and individualize student responses by including multisensory modes of input that may or may not include technology for instructional materials, student response modes, and instructional activities.

*Evidence*

Unfortunately, limited research was found to be conducted with students with sensory or physical impairments; however, it is not assumed that the students did not have sensory or physical impairments. For example, students with autism spectrum disorder may have sensory impairments, and students with cerebral palsy may have physical impairments that may create obstacles for access without addressing these needs when planning for instruction. Four studies specifically mentioned the students had physical impairments, and two studies specifically mentioned participants with sensory impairments. Within those research studies, participants may have been classified as having a sensory impairment if labeled as visually impaired even if it was noted the students did not need additional material modifications. Developing math materials for students with limited physical mobility, blind, deaf, or other sensory issues can be challenging for teachers. When given information about and concrete examples of the three components of UDL (expression, representation, engagement) teachers were able to create lessons plans that accessed general curriculum goals that also meet individual needs of the students (Spooner, Baker, Harris, Ahlgrim-Delzell, & Browder, 2007). In order for students with sensory or physical impairments to have equal access to the general education curriculum, research is needed in these two areas to provide teachers with guidance for instructional planning to address the sensory and physical needs of students with sensory and physical impairments.

## Summary

Clearly, more research in needed in the area of mathematics instruction for students with significant cognitive disabilities, including individuals who have physical and/or sensory impairments. Despite these limitations, this review of the instructional practices in mathematics can serve as a guide for teachers of students with significant cognitive disabilities. From this and prior reviews, teachers can be sure of these overarching themes: (a) systematic instruction is an evidence-based practice for teaching mathematics to students with significant cognitive disabilities; (b) there is strong support for teaching in the context of the community; functional applications will ensure the skills are personally relevant and generalize to other settings; (c) teachers must consider appropriate prompting strategies depending on the targeted math skill, and prompts must fade over time for independence; (d) the general education teacher can assist with the challenging task of aligning instruction to grade levels in all strand of math; (e) students can learn in a general education setting using the concept of UDL and/or the strategy of embedded instruction; (f) assistive technology and other types of technology should be considered to ensure participation for all students, (g) all four phases of learning should be addressed when teaching a math skill (i.e., acquisition, fluency, generalization, and maintenance); and (h) considerations for students with sensory and physical impairments using UDL should be made in the design of accessible curricula.

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* Denotes studies included in the literature review

Table 1: *Summary of the fourteen studies that meet the inclusion criteria*

| Reference | Independent Variable/ instructional Strategy | Participants * | Math strand | Dependent variable/ targeted skill | Setting/ who taught | Results (PNDs) |
|---|---|---|---|---|---|---|
| Ayres et al. (2006) | Systematic instruction, massed trials; simultaneous prompting | 4 (3 males, 1 female); age 14; HI | Measurement | ID next dollar for purchasing | Special education setting; computer program | 49% |
| Akmanoglu & Batu (2004) | Systematic instruction, massed trials; simultaneous prompting | 2 males; age 12 & 17; AU | Numbers and Operations | pointing to #s 1-9 | Special education setting; researcher | 68.28% |
| Cihak et al. (2006) | Systematic instruction, total task chaining; most-to-least hierarchy | 6 males; age range 11-15; IQ range 36-51 | Measurement, Algebra, & Numbers and Operations | purchasing & banking skills | Special education classroom and community setting; special education teacher | 94.4% pictures<br>100% videos |
| Cihak & Foust (2008) | Systematic instruction, total task chaining; least –to-most hierarchy | 3 (1 male, 2 females); age 7-8; AU | Numbers and Operations | single-digit addition problem solving skills using number & touch-point strategies | Special education classroom; special education teacher | 46% number line<br>96% touch points |
| Cihak & Grim (2008) | Systematic instruction, massed trials; most-to-least hierarchy | 4 (2 males, 2 females); age 15-17; IQ range 35-50 | Measurement & Numbers and Operations | purchasing with counting on and next dollar | Special education classroom; special education teacher | 97.9% |
| Collins et al. (2007) | Systematic Instruction, milieu teaching, massed, distributed and embedded instruction; simultaneous prompting | 2 (1 male, 1 female); age 13; PI, VI; IQ 40 & 43. | Measurement, Geometry, & Algebra | Math sight words (point, quart, gallon, acute, exponent, integer) | General education setting; special education, general education teacher, and paraprofessional | 81% function word<br>79.5% core word |
| DiPipi-Hoy & Jitendra (2007) | Systematic instruction, total task chaining; CTD | 2 females; age 16-20 | Measurement & Numbers and Operations | purchasing skills | Community; parent | 100% |

| Reference | Independent Variable/ instructional Strategy | Participants * | Math strand | Dependent variable/ targeted skill | Setting/ who taught | Results (PNDs) |
|---|---|---|---|---|---|---|
| Falkenstine et al. (2009) | Systematic instruction, total task chaining, massed trials; CTD | 1 female; age 16; PI; IQ 42 | Measurement & Numbers and Operations | time telling | Special education classroom; special education teacher | 76.5% |
| Hansen & Morgan (2008) | Systematic instruction, total task chaining; simultaneous prompting | 3 females; age 16-17; IQ range 45-55 | Measurement | using money to purchase grocery store items | school computer lab; computer program | 100% |
| Jimenez et al. (2008) | Systematic instruction, total task chaining; least-to-most hierarchy and constant time delay | 3 (2 males, 1 female); age 15-17; IQ range 41-49 | Algebra | solving a linear equation | Special education classroom & general education classroom; special education teacher & peer | 99.2% |
| Mechling et al. (2008) | Systematic instruction, total task chaining; most-to-least hierarchy | 3 (2 males, 1 female); age 19-22; IQs 40 & 53 | Measurement & Numbers and Operations | dvd to teach cooking (press number “3” on microwave, stir 6 times, measure ¾ cup) | kitchen in rented apt for home skills ; researcher | 100% |
| Neef et al. (2003) | Systematic instruction, massed trials; simultaneous prompting | 1 male; age 19; IQ 46 | Algebra & Numbers and Operations | solving addition & subtraction story problems | Special education classroom; researcher | 100% |
| Polychronis (2004) | Systematic Instruction, milieu teaching, and embedded instruction; least-to-most hierarchy | 2 males; age 7 & 11); VI; PI , IQ 45 (one not given) | Measurement and Numbers and Operations | number identification 1-9 & time telling | General education setting; general education teacher | 100% 30 min trial<br>98.4% 120 min trial |
| Rao & Mallow (2009) | Systematic instruction, massed trials; simultaneous prompting | female; 7$^{th}$ grade; PI; IQ 49 | Numbers and Operations | multiplication facts | Special education classroom; special education teacher | 100% |

* VI = visual impairment, PI = physical impairment, HI = hearing impairment, AU = Autism

Annotated Bibliography

**Akmanoglu, N., & Batu, S. (2004). Teaching pointing to numerals to individuals with autism using simultaneous prompting. *Education and Training in Developmental Disabilities, 39*, 326-336.**

National math strand covered in this study is numbers/operations. The target skill for intervention is pointing to numbers from 1 through 9. Instructional strategies include systematic instruction through massed trials with the use of simultaneous prompting. Student participants in receipt of the intervention are 2 males aged 12 and 17 with autism. Delivery of instruction is by the researcher, and location of instruction is in a special education setting. Number recognition is an important skill, but teaching number identification in a personally relevant context (e.g., date, phone number) instead of teaching in isolation may be more useful for students because it offers an opportunity to use the skill in a real-world context.

**Ayres, K. M., Langone, J., Boon, R. T., & Norman, A. (2006). Computer-based instruction for purchasing skills. *Education and Training in Developmental Disabilities, 41*, 253-263.**

National math strand covered in this study is measurement. The target skill for intervention is identification of the next dollar for use in making purchase. Instructional strategies include systematic instruction through total task chaining with the use of simultaneous prompting. Student participants in receipt of the intervention are one female and three males in middle school with intellectual disabilities. Delivery of instruction is by a computer program, and location of instruction is in a special education setting. Purchasing skills are essential to developing independence; however, the setting of this study is limited to special education environment. In order for students to apply this skill, instruction or practice should include in-situ settings such as a school cafeteria or a store in the community.

**Cihak, D., Alberto, P. A., Taber-Doughty, T., & Gama, R. I. (2006). A comparision of static picture prompting and video prompting simulation strategies using group instructional procedures. *Focus on Autism and Other Developmental Disabilities, 21*, 89-99.**

National math strands covered in this study are numbers/operations, measurement, and algebra. The target skills for intervention are banking and purchasing skills. Instructional strategies include systematic instruction through total task chaining with the use of a most-to-least prompting hierarchy. The student participants in receipt of intervention are 6 males in the age range of 11-15 with an IQ range of 36-51. Delivery of instruction is by the special education teacher, and location of instruction is in the special education setting and the community. Students need ample practice to generalize skills they are learning or skills they acquired in the classroom to natural contexts, and this study provides an example of planning for instruction in the classroom and the community for educators to reference.

**Cihak, D. F., & Foust, J. L. (2008). Comparing number lines and touch points to teach addition facts to students with autism. *Focus on Autism and Other Developmental Disabilities, 23*, 131-137.**

National math strands covered in this study is numbers/operations. The target skill for intervention is solving single-digit addition problems by using a number line or touch points. This study compares the use of number lines and touch points for effectiveness in solving single-digit addition problem. Instructional strategies include systematic instruction through total task chaining with the use of a least-to-most prompting hierarchy. Student participants in receipt of intervention are 2 females and a male in the age range of 7-8 with autism. Delivery of instruction is by the special education teacher, and location of instruction is in the special education setting. Students improved their ability to solve single-digit addition problems by using number lines and touch points, but outcome of this study found touch points to be more effective than number lines. However, the study does not address the fading of the touch points. In order to validate findings, replication with more students is necessary. It may also be noteworthy to consider that the students in the study had autism, and outcomes may differ with students that have disabilities.

**Cihak, D., & Grim, J. (2008). Teaching students with autism spectrum disorder and moderate intellectual disabilities to use counting-on strategies to enhance independent purchasing skills. *Research in Autism Spectrum Disorders, 2*, 716-727.**

National math strands covered in this study is numbers/operations and measurement. The target skill for intervention is purchasing with counting on and next dollar. Instructional strategies include systematic instruction through massed trials with the use of a most-to-least prompting hierarchy. Student participants in receipt of intervention are 2 females and 2 males in the age range of 15-17 with an IQ range of 35-50. Delivery of instruction is by the special education teacher, and location of instruction is in the special education setting. The proficient use of counting on and next dollar strategy for making purchases may help students to develop autonomy; but the setting of this study is limited to special education environment. Restricting instruction to the classroom does not allow students to gain practice using strategies within context, and this restriction may hinder the development of autonomy.

**Collins, B. C., Evans, A., Creech-Galloway, C., Karl, J., & Miller, A. (2007). Comparison of the acquisition and maintenance of teaching functional and core content sight words in special and education setting. *Focus on Autism and Other Developmental Disabilities, 4,* 220-233.**

National math strands covered in this study are measurement, geometry, and algebra. The target skill for intervention is acquisition of math sight words relevant to content (e.g., gallon, acute, integer). Instructional strategies include systematic instruction through milieu teaching with massed, distributed, and embedded trials with the use of simultaneous prompting. Student participants in receipt of intervention are a female and a male. Both students are 13 years old with the IQs of 43 and 40 respectively. Delivery of instruction is by the special education teacher, and the general education teacher, and the paraprofessional. Location of instruction is in the general education setting. This study exemplifies effective instructional strategies for academic content alignment to use in the general education with students that have severe disabilities.

**DiPipi-Hoy, C., & Jitendra, A. (2004). A parent-delievered intervention to teach purchasing skills to young adults with disabilities. *The Journal of Special Education, 38,* 144-157.**

The national math strands covered in this study are numbers/operations and measurement. The target skill for intervention is purchasing skills. The instructional strategies include systematic instruction through total task training with a constant time delay prompting system. The student participants in receipt of intervention are 2 females age 16 and 20 years of age. Delivery of instruction is by parents and, location of instruction is in the community. Students with severe disabilities need opportunities to generalize skills to different setting and with different people, including their parents. This study provides a reference for teachers to when planning for students to learn skills or generalize skills to the community with their parents.

**Falkenstine, K. J., Collins, B. C., Schuster J. W., & Kleinert, H. (2009). Presenting chained and discrete tasks as non-targeted information when teaching discrete academic skills through small group instruction. *Education and Training in Developmental Disabilities, 44,* 127-142.**

National math strands covered in this study are numbers/operations and measurement. The target skill for intervention is telling time. Instructional strategies include systematic instruction through total task training with a constant time delay prompting system. Student participant in receipt of intervention is a 16-year old female with an IQ of 42. Delivery of instruction is by the special education teacher, and location of instruction is the special education classroom. The ability to tell time helps students to develop skills necessary for independence and self-determination because it allows increased opportunities for students to engage in activities such decision making and problem solving. It is important that students do not learn the skill of telling time in isolation; students need practice telling time with activities such as setting an alarm clock or developing a schedule for functional application.

**Hansen, D., & Morgan, R. (2008). Teaching grocery store purchasing skills to students with intellectual disabilities using a computer-based instruction program. *Education and Training in Developmental Disabilities, 43,* 431-442.**

National math strand covered in this study is measurement. The target skill for intervention is using money to purchase items in a grocery store. Instructional strategies include systematic instruction through total task training with simultaneous prompting. Student participants in receipt of intervention are 3 females in the age range of 16-17 with an IQ range of 45-55. Delivery of instruction is by a computer program, and the location of instruction is the computer lab. Educators need to have updated knowledge about combining technology with instruction. The use of computer programs to deliver instruction in the setting of a computer lab allows differentiated instruction among students with and without disabilities to occur in one classroom.

**Jimenez, B. A., Browder, D. M., Courtade, G. R. (2008). Teaching an algebraic equation to high school students with moderate developmental disabilities. *Education and Training in Developmental Disabilities, 43,* 266-274.**

National math strand covered in this study is algebra. The target skill for intervention is solving linear equations. Instructional strategies include systematic instruction through total task training with prompting systems of constant time delay in a least-to-most prompt hierarchy. Student participants in receipt of intervention are 1 female and 3 males in the age range of 15-17 with an IQ range of 41-40. Delivery of instruction is the special education teacher and peer. Location of instruction is the special education classroom and the general education classroom. Teachers should not restrict mathematical instruction exclusively to the special education classroom. This study gives teachers guide for instructional planning for students to generalize acquired skills learned in the special education class to the general education class with assistance of peers.

**Mechling, L. C., Gast, D. L., & Fields, E. (2008). Evaluation of a portable DVD player and system of least prompts to self-prompt cooking task completion by young adults with moderate intellectual disabilities. *Journal of Special Education*, *42*, 179-190.**

National math strand covered in this study is numbers/operations and measurement. The target skill for intervention is self-prompting with a DVD to teach math skills related to cooking such as pressing appropriate numbers on the microwave. Instructional strategies include systematic instruction through total task training with a most-to-least prompt hierarchy. Student participants in receipt of intervention are 1 female and 2 males in the age range of 19-22 with an IQ range of 40-53. Delivery of instruction is the researcher, and location of instruction is the kitchen of a rented apartment that the school uses to teach skills for home living. This study provides in-situ instruction for home living skills for students to practice these skills in context. This is important for functional application, and it is essential for ameliorating potential safety issues for skills such as cooking.

**Neef, N. A., Nelles, D. E., Iwata, B. A., & Page, T. P. (2003). Analysis of precurrent skills in solving mathematics story problems. *Journal of Applied Behavior Analysis*, *36*, 21-33.**

National math strands covered in this study are numbers/operations and algebra. The target skill for intervention is solving story problems by using addition or subtraction. Instructional strategies include systematic instruction through massed trials with simultaneous prompting. Student participant in receipt of intervention is a male 19-years old with an IQ of 45. Delivery of instruction is by the researcher, and location of instruction is the special education classroom. Most special education teachers consider algebra a higher level strand of mathematics, and the research for teaching higher level strands of mathematics to this population is limited. The positive outcomes of this study contribute to the evidence regarding the ability of students with severe disabilities to learn higher strands of mathematics.

**Polychronis, S. C., McDonnell, J., Johnson, J. W., Riesen, T., & Jameson, M. (2004). A comparison of two trial distribution schedules in embedded instruction. *Focus on Autism and Other Developmental Disabilities*, *19*, 140-151.**

National math strands covered in this study is numbers/operations and measurement. The target skill for intervention is number identification from 1 to 9 with telling time. Instructional strategies include systematic instruction through milieu teaching and embedded trials with a least

least–to-most prompt hierarchy. Student participants in receipt of intervention are 2 males, 7 and 11 years of age. One of the participants has an IQ of 49, but the IQ of the other participant is not given. Delivery of instruction is by the general education teacher, and location of instruction is the general education classroom. This study teaches students to identify numbers within the context of telling time instead of teaching number identification in isolation. The students learn to tell time with their nondisabled peers in the general education classroom, so academic instruction for all students occurs in one classroom setting.

**Rao, S. & Mallow, L. (2009). Using simultaneous prompting procedure to promote recall of multiplication facts by middle school students with cognitive impairment. *Education and Training in Developmental Disabilities, 44,* 80-90.**

National math strand covered in this study is numbers/operations. The target skill for intervention is recall of multiplication facts. Instructional strategies include systematic instruction through massed trials with simultaneous prompting. Student participant in receipt of intervention is a female in 7th grade with an IQ of 49. Delivery of instruction is by the special education teacher, and location of instruction is the special education classroom. Fluent and accurate computation provides students with increased opportunities for autonomy because computation skills are a key component of life skills mathematics. However, mathematic instruction needs to reach beyond the classroom for students to access the skills in real-life settings.